



ABS Education Services

Name: _____

CaSQ 9B

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The Theory of the Birth Month Paradox

How to Get a Random Sample from CensusAtSchool

Go to the CensusAtSchool [Random Sampler](#) to download a sample.X

Reference year: *(select year)*

Sample size: *35 students*

Select questions: *Birth month*

Location: *Select location*

Year level:

(select a range of year levels)

To protect privacy there is a rule built into the sampler that the requested sample size cannot exceed 10% entered.

Task:

If you completed Worksheet 9A you would have found that for any 5 people, chosen at random, the probability that at least 2 of the 5 will have the same birth month is more than 50%.

This result was reached using a long run experiment where results were compiled after collecting data over many trials.

Experimental probability is mainly used when it is not possible to work out the theoretical probability using the mathematical formula below:

$$\text{Pr}(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

The result of the Birth Month paradox can be checked using theoretical probability. However, it can be explained more easily by finding the probability that no 2 birth months will be the same.

Complementary events in probability are events that cannot occur together (e.g. when tossing a coin, obtaining a head is complementary to tossing a tail). The probabilities of complementary events add to 1 i.e. $\text{Pr}(\text{Event}) + \text{Pr}(\text{Complement of the Event}) = 1$

So,

$$\text{Pr}(\text{Event}) = 1 - \text{Pr}(\text{its Complementary Event})$$

So we can say for 5 people chosen at random:

$$\text{Pr}(\text{at least 2 birth months will be the same}) = 1 - \text{Pr}(\text{none are the same})$$

CensusAtSchool Random Sampler <http://www.cas.abs.gov.au/cgi-local/cassampler.pl>

Access other similar lessons at <http://www.abs.gov.au/censusatschool> CensusAtSchool Random Sampler <http://www.cas.abs.gov.au/cgi-local/cassampler.pl>

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Pr (Person 1 has a birth month) = $\frac{12}{12}$ *any month can be chosen*

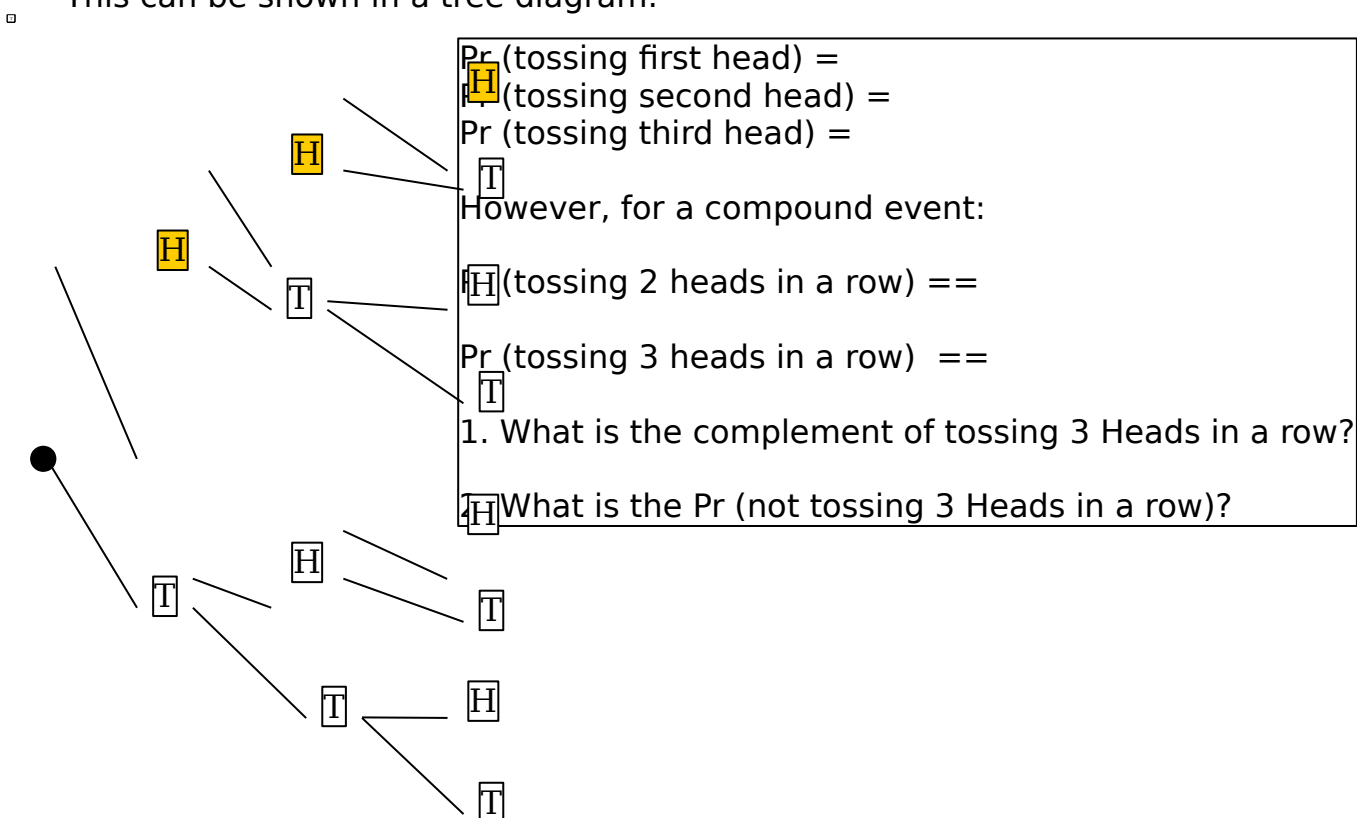
Pr (Person 2 is different to Person 1) = $\frac{11}{12}$ *any of the remaining 11 months is ok*

Pr (Person 3 is different to Persons 1 & 2) = $\frac{10}{12}$ *can choose any of the 10 months left*

The birth month paradox is an example of a compound event. This means that there are a number of steps in the event. Another example of a compound event is tossing a coin 3 times.

1. What is the Pr (H, H, H) if we toss a coin 3 times?

This can be shown in a tree diagram.



We can apply this multiplication rule to the birth month paradox.



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The Pr (3 people each has a different birth month) = $\frac{12}{12} \times \frac{11}{12} \times \frac{10}{12}$

2. Continue this pattern to find Pr (5 people have different birth months).

3. Use your understanding of complementary events to find, for 5 people chosen at random, Pr (not all birth months will be different). Is this the same as your answer for 9A?

Extension:

Similarly in the birthday paradox:

For every 23 people randomly selected there is a 50% chance that at least 2 will share the same birthday, If there are 30 then there is a 70% probability and if there are 50 the probability rises to 97%.

4. Can you show the theory behind these results?